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Author(s):	Langenbrunner, James R. Booker, Jane M.
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Author(s):

James R. Langenbrunner, Los Alamos National Laboratory, XCP-8

Jane M. Booker, Booker Scientific Consulting

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POWER INDEX TABLES FOR BURNING D-T AND D-D PLASMA

James R. Langenbrunner & Jane M. Booker

Los Alamos National Laboratory
XCP-8, Mail Stop F644
Los Alamos, New Mexico 87545

Abstract

The thermonuclear fusion reactivities for D-T and D-D can be expressed as a power law of ion temperature (keV). It will be shown that the product of ion temperature and the ratio of the first derivative of the reactivity to the reactivity is the power index. Tabled values of the power index, as a function of ion temperature, *T,* are presented for reference. A connection to gamma diagnostics for fusion is briefly introduced.

Introduction

Prompt Diagnostics (PD) of nuclear-fusion reactions comprise neutron and gamma-ray diagnostics, spatially or time-resolved, or both. Fusion reactions have both neutron and gamma branches. The branching ratios are constant over the energies that are typically accessed in hydrodynamic fusion experiments.

Hydrodynamic fusion experiments, in contrast with quasi-monochromatic nuclear-beam experiments, are complicated to understand, and require more knowledge. Hydrodynamic experiments of interest to fusion are frequently driven by strong shock waves driving compression. These require, if detailed understanding is desired, knowledge of the interacting volume and density of reactants, and possibly details of instabilities that could develop. Such quantities are of course changing quickly as a function of time. It is for that reason, that initially, a PD of choice is the gamma diagnostic. This is because, at least, the sequence of events is unambiguous: information reading the product reaction-rate arrives at the detector in the same sequence in time that the gammas are created. The time-of-flight for gammas is constant regardless of energy. This makes a comparison with simulation for reaction history easier using gamma rays, compared with neutrons.

Two rarely occurring (~ 1 in 10^5 reactions) deuterium-tritium (D-T) reactions branches and one rarely occurring (~ 1 in 10^6 reactions) deuterium-deuterium (D-D) reaction branch produce γ-rays with energies in the 20 MeV range. D-T fusion produces an excited ^{5}He nucleus, which de-excites via two high-energy gamma branches, while D-D capture fusion goes directly to the ^{4}He ground state. These gammas can be measured in "current-mode" using, for example, gas Cherenkov γ-ray detectors with fast temporal responses and inherent energy thresholds. The equivalent measure of flux of the γ-rays, $\overset{\cdot}{\gamma}$, is proportional to the reaction rate, which in turn, depends upon the temperature, if the temperature can be defined. Clues to the hydrodynamic sequence of events associated with compression can be found by studying the gamma-ray alpha, α, the logarithmic time-derivative of gamma flux,:

$$\alpha = \frac{\dfrac{d\overset{*}{\gamma}}{dt}}{\overset{*}{\gamma}} = \frac{d\ln\overset{*}{\gamma}}{dt} \tag{1}$$

It will be shown, below, that α can also be defined in terms of the power index *a(T)* introduced here:

$$\alpha = \frac{a(T)}{T}\frac{dT}{dt} . \tag{2}$$

where ion temperature, *T,* is in keV.

In practice, the fusion reactivity, <σ*v*>, is commonly expressed as the average over a Maxwellian velocity distribution, *g(v)*:

$$<\sigma v> = \int_0^\infty \sigma(v)\cdot v\cdot g(v)dv = \int_0^\infty \sigma(v)\cdot v\cdot 4\pi\cdot\left(\frac{m}{2\pi T}\right)^{1.5}\cdot v^2\cdot\exp\left(\frac{-mv^2}{2T}\right)\cdot dv \tag{3}$$

where reactivity has units of cm^3/sec.

Various formulations for calculating (3) have been popularized, such as U.S. Naval Research Lab (NRL) [1] formulary, Hively [2], Caughlan & Fowler [3] and Bosch & Hale [4]. A comparison study [5] of these and other calculations for D-D and D-T reactions demonstrated the favorable performance of Bosch & Hale, which is the chosen

method here.

Reactivity as a Power Law of Ion Temperature

Reactivities have been explicitly expressed as a power law of ion temperature, *T*, in [6] & [7]. In [6], the notation of <σ*v*>=*f*(*T*) was adopted to emphasize the relationship with *T*, and that is continued in this report. Reactivity then becomes: $< \sigma v > \equiv f(T) \sim T^a$:

$$< \sigma v > \equiv f(T) \cong T^a \tag{4}$$

where *a* is not necessarily an integer and, as will be seen below, can vary with *T*. An example below illustrates the role of *a*.

Using the NRL D-T reactivity formulation [1], the authors demonstrated [7] that *f*(*T*) was accurately fitted with integer *a*'s for a range of *T*'s from 0.1-10 keV as follows:

Table I. NRL D-T Reactivity Fits for 0.1≤*T*≤10 keV from [7].

T range (keV)	*a*
0.1-1	6
1-2	5
2-3	4
3-10	3

These reactivity fits were precise enough to accommodate 99.9% to 99.99% of the variability in reactivity. The decreasing trend in *a* with *T*, signaled further study into the power law behavior of *f*(*T*) using other formulations.

Bosch & Hale Reactivity Formulation

One of the most prominently cited formulations for reactivity is from Bosch & Hale [4]. Equation (5) with its coefficient table contains their new, improved, parameterization:

$$< \sigma v >= f(T) = \frac{C_1 \xi^2 \exp(-3\theta^{1/3}\xi)}{\theta^{5/6}}$$

$$\theta = 1 - \frac{(C_2 T + C_4 T^2 + C_6 T^3)}{(1 + C_3 T + C_5 T^2 + C_7 T^3)} \tag{5}$$

$$\xi = \frac{C_0}{T^{1/3}} \quad .$$

Parameters	D+D—>3He+n	D+D—>T + p	D-T
C_0	6.2696	6.2696	6.661
C_1	3.57E-16	3.72E-16	6.43E-14
C_2	5.86E-03	3.41E-03	1.51E-02
C_3	7.68E-03	1.99E-03	7.52E-02
C_4	0	0	4.61E-03
C_5	-2.96E-06	1.05E-05	1.35E-02
C_6	0	0	-1.07E-04
C_7	0	0	1.37E-05

Based upon the complexity in (5), one might expect a high degree of accuracy over a wide range of T, and that is the experience of studies by the authors [5] and of Horny′ et al. [8].

The complexity of Bosch & Hale in (5) makes it difficult to determine the first analytical derivative of $f(T)$ with T, f', which is given in (6). For simplification, the first derivative is expressed in terms of $f(T)$, θ, ξ, $d\theta/dT$ and $d\xi/dT$:

$$f' = \frac{df(T)}{dT} = \frac{-5f(T)}{6\theta}\frac{d\theta}{dT} - \frac{2f(T)}{3T} + \frac{f(T)}{T}\left[\frac{-C_1 T^{2/3}}{\theta^{2/3}}\frac{d\theta}{dT} - \xi\theta^{1/3}\right] \tag{6}$$

where

$$\frac{d\theta}{dT} = \frac{-[(\theta-1)(C_3 + 2C_5 T + 3C_7 T^2) + (C_2 + 2C_4 T + 3C_6 T^2)]}{(1 + C_3 T + C_5 T^2 + C_7 T^3)} \quad .$$

Power Index Development

Taking the derivative of reactivity with T in the power law expression in (4), gives the approximated determination for a.

$$f' = \frac{df(T)}{dT} \cong aT^{(a-1)} = \frac{a}{T}T^a = a\frac{f(T)}{T} \quad . \tag{7}$$

Temporarily assigning a proportionality factor, k, converts the approximation sign in (7) to an equality as:

$$f' = ka\frac{f(T)}{T} \quad . \tag{8}$$

The power index factor, $a(T)$, is defined as the product of that constant and the power exponent a. It is further generalized by including the dependence upon T. The first derivative is now an expression for the power index:

$$\frac{d}{dT}f(T) = a(T)\frac{f(T)}{T} \ .$$
(9)

Rearranging (9), $a(T)$ becomes the product of ion temperature and the ratio of the first derivative of the reactivity to the reactivity:

$$a(T) = T\frac{f'}{f(T)} \ .$$
(10)

This line of thought is extended to describe the slope of alpha with time, in Ref. [9].

Bosch & Hale Power Index Values

Determining the power index of the Bosch & Hale reactivity requires some algebraic manipulation to conform (6) to the expression in (9). Note that the temperature derivative of $f(T)$ in (6) has reactivity in every term and a factor of $1/T$ in almost every term. The form for determining $a(T)$ is accomplished by rearranging the definition of θ in terms of a factor for T

$$\theta = T\frac{(T^{-1}+C_3-C_2+(C_5-C_4)T+(C_7-C_6)T^2)}{(1+C_3T+C_5T^2+C_7T^3)} = \frac{T}{q}$$
(11)

and substituting (11) for θ into the denominator of the first term on the right side of (6). Thus f' is defined according to (10):

$$f' = \left(\frac{-5q}{6}\frac{d\theta}{dT} - \frac{2}{3} + \left[\frac{-C_1T^{2/3}}{\theta^{2/3}}\frac{d\theta}{dT} - \xi\theta^{1/3}\right]\right)\frac{f(T)}{T}$$
(12)

The Bosch & Hale $a(T)$ is:

$$a(T) = \frac{-5q}{6}\frac{d\theta}{dT} - \frac{2}{3} + \left[\frac{-C_1T^{2/3}}{\theta^{2/3}}\frac{d\theta}{dT} - \xi\theta^{1/3}\right] \ .$$
(13)

Values of the power index, $a(T)$, for the range of ion temperatures from 0.3 to 25 keV are presented for reference in Table II. Two D-D branches are listed separately along with the D-D total. It should be noted that branch D-D reacitivies and their first derivatives add prior to calculating the power index in (13).

Table II. Bosch & Hale Power Index Values for DT and DD for $0.3 \leq T \leq 25$ keV.

T (keV)	$a(T)$ D+D—>³He+n	$a(T)$ D+D—>T + p	$a(T)$ D-D total	$a(T)$ D-T
0.3	8.71	8.71	8.71	9.32
0.4	7.85	7.86	7.85	8.42
0.5	7.24	7.25	7.25	7.79
0.6	6.78	6.79	6.78	7.30
0.7	6.41	6.42	6.41	6.92
0.8	6.1	6.11	6.11	6.60
0.9	5.84	5.85	5.85	6.34
1	5.62	5.63	5.63	6.11
1.1	5.43	5.44	5.43	5.91
1.2	5.25	5.27	5.26	5.74
1.3	5.1	5.11	5.11	5.58
1.4	4.96	4.97	4.97	5.44
1.5	4.83	4.85	4.84	5.32
1.6	4.72	4.73	4.73	5.20
1.7	4.61	4.63	4.62	5.10
1.8	4.51	4.53	4.52	5.00
1.9	4.42	4.44	4.43	4.92
2	4.34	4.36	4.35	4.83
2.1	4.26	4.28	4.27	4.76
2.2	4.18	4.2	4.19	4.68
2.3	4.11	4.14	4.12	4.62
2.4	4.05	4.07	4.06	4.55
2.5	3.99	4.01	4	4.49
2.6	3.93	3.95	3.94	4.44
2.7	3.87	3.89	3.88	4.38
2.8	3.82	3.84	3.83	4.33
2.9	3.77	3.79	3.78	4.28
3	3.72	3.74	3.73	4.24
3.1	3.67	3.7	3.68	4.19
3.2	3.63	3.65	3.64	4.15
3.3	3.59	3.61	3.6	4.11

3.4	3.54	3.57	3.56	4.07
3.5	3.51	3.53	3.52	4.03
3.6	3.47	3.5	3.48	3.99
3.7	3.43	3.46	3.45	3.96
3.8	3.4	3.43	3.41	3.92
3.9	3.36	3.39	3.38	3.89
4	3.33	3.36	3.34	3.85
5	3.06	3.09	3.07	3.56
6	2.85	2.89	2.87	3.31
7	2.68	2.72	2.7	3.08
8	2.55	2.59	2.57	2.88
9	2.43	2.48	2.46	2.69
10	2.34	2.39	2.36	2.51
11	2.25	2.31	2.28	2.35
12	2.18	2.23	2.21	2.21
13	2.11	2.17	2.14	2.07
14	2.05	2.11	2.08	1.94
15	2	2.06	2.03	1.83
16	1.95	2.01	1.98	1.72
17	1.91	1.97	1.94	1.62
18	1.87	1.93	1.9	1.53
19	1.83	1.9	1.87	1.44
20	1.8	1.86	1.83	1.36
21	1.77	1.83	1.8	1.29
22	1.74	1.8	1.77	1.22
23	1.71	1.78	1.74	1.15
24	1.68	1.75	1.72	1.09
25	1.66	1.73	1.69	1.04

Figure 1 shows how the D-T power index differs from the one for the total D-D.
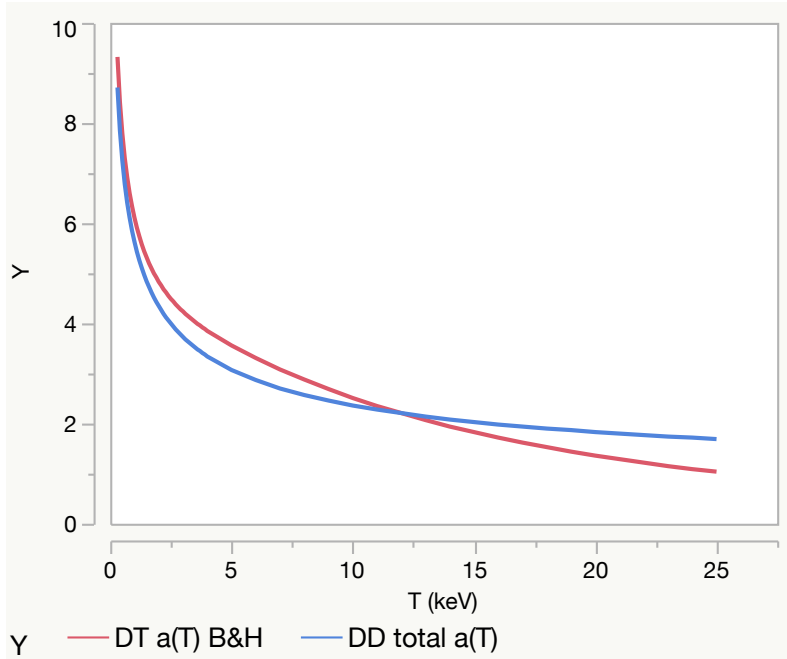
Figure 1. Power index (Y) plotted against ion temperature for Bosch & Hale D-T (red) and total (both branches) D-D (blue).

The power indices for D-D n branch, D-D p branch and D-D total are close to each and close to D-D total, as seen in Figure 2. Figure 3 shows how the Bosch & Hale power index, $a(T)$, aligns with the power, a, for T given in Table I. The blue segments reflect the course intervals of T from that table. In addition, differences between the NRL formulation of reactivity and the Bosch & Hale reactivity can be detected with the latter curve positioned above the segments. Even accounting for formulation differences and granularity, there is a correspondence between the power index, $a(T)$, and the general power law for T in (4).
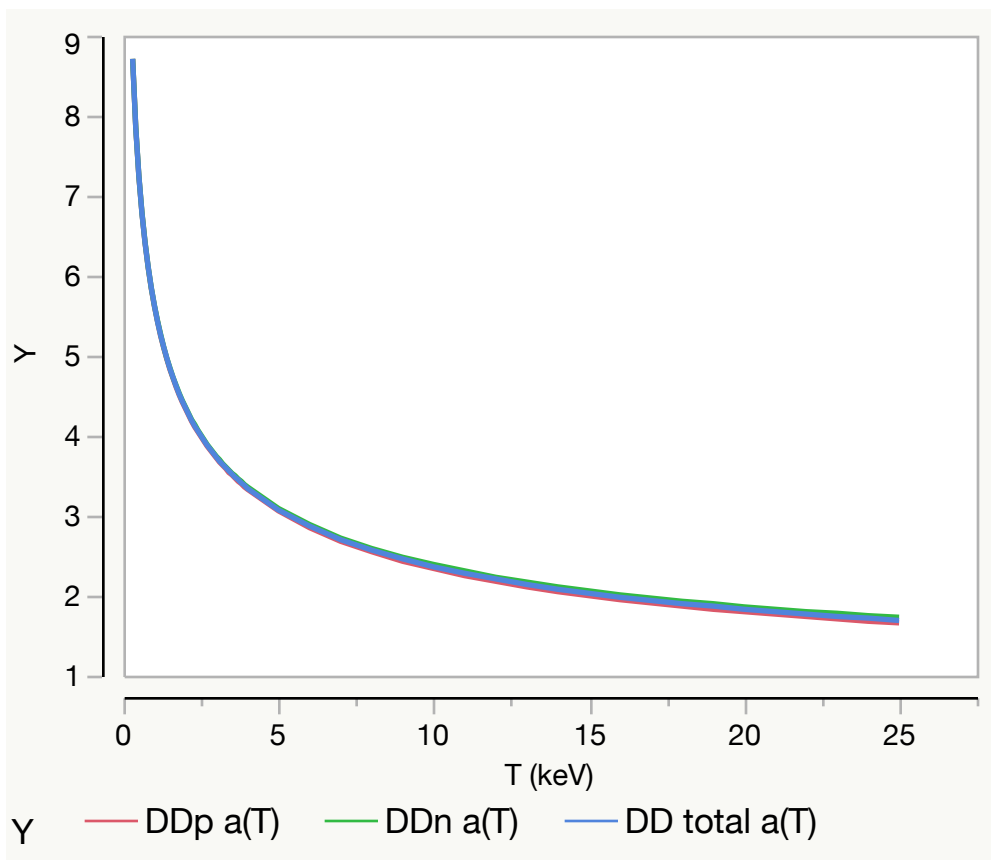
Figure 2. Power index (Y) plotted against ion temperature for Bosch & Hale D-D p (red), D-D n (green) and total p D-D (blue).
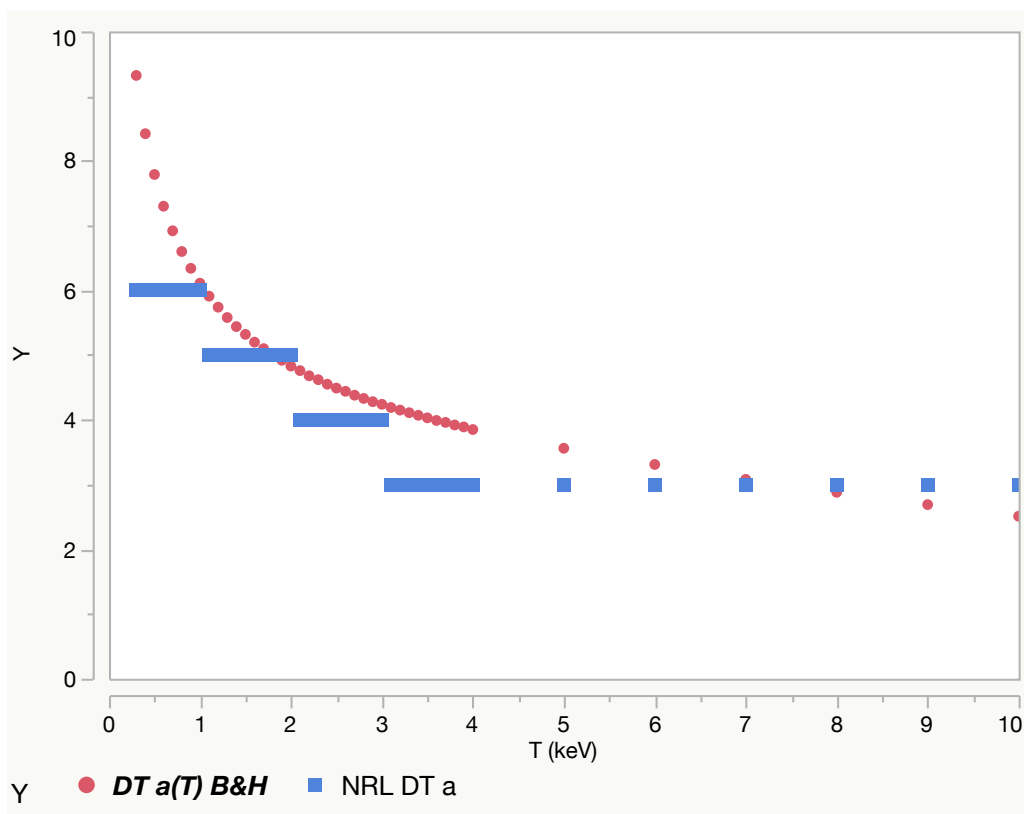
Figure 3. Bosch & Hale D-T power index (red) compared to NRL power law for *T* (blue segments).

Closing Remarks

The present research is undertaken as a means of improving verification and validation. For example, it is common for users of the NRL formulary to cite a power index. It is not as common for the power index to be cited using Bosch & Hale [4], which has a more valid and more complete physics basis. Also, it is not so common to see the difference in the $D + D \longrightarrow {}^3He + n$ and the $D + D \longrightarrow T + p$ branches explicitly calculated, and this is done herein, specifically in Table II. A description of the energy dependent analysis of the R-matrix code, which is used as the physics basis for the Bosch and Hale rates, was provided to the authors [10].

The NRL Formulary reaction rates are known to be inaccurate over certain temperature ranges. It is recommended that one use the Bosch & Hale rates where plasma screening effects are not significant. In the case of non-negligible plasma screening, there is an additional enhancement to reactivity. The authors would like to thank Mark Paris and Brian Albright for bringing this topic to the authors for future discussion, [11].

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